

EXAM REVIEW II

TUESDAY DECEMBER 10

$$a_1 + a_2 + \dots + a_n = \frac{(a_1 + a_n) * n}{2}$$

```

1 void prog(int[] a, int n)
2   for (int i = 0; i < n; i++) {
3     for (int j = i; j < n; j++) {
4       for (int k = j; k > 0; k--) {
5         System.out.println(i * j + k);
6       }
7     }
8   }

```

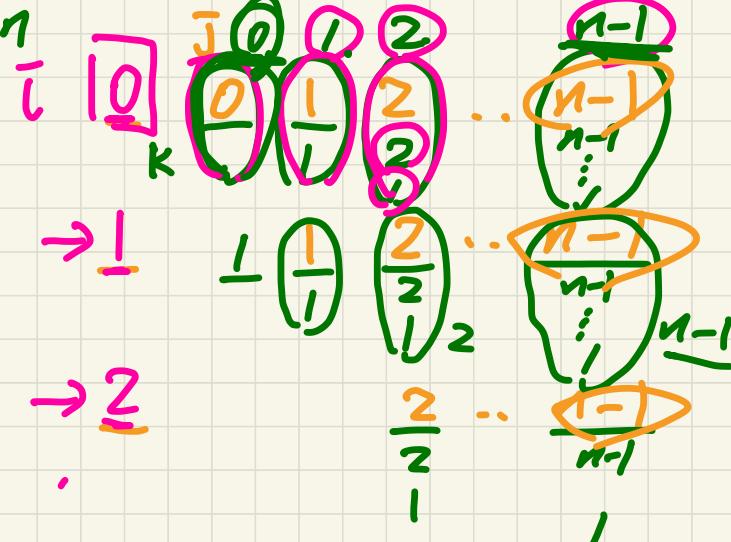
$\sum_{i=0}^{n-1} \frac{((i + (n-1)) * (n-i))}{2}$

body of loop

$i=0 \quad \frac{(0 + (n-1)) * (n-1)}{2}$

$i=1 \quad \frac{(1 + (n-1)) * (n-1)}{2}$

$i=n-1 \quad \frac{((n-1) + (n-1)) * (n-(n-1))}{2}$



$\rightarrow 1$

$\rightarrow 2$

:

:

$\rightarrow n-1$

$RT = O(1 * \# \text{ of iterations})$

\downarrow

each iteration

$$\frac{n-1}{n-1}$$

Exercise -

```
1 void prog(int[] a, int n)
2     for (int i = 0; i < n; i++) {
3         for (int j = i; j < n; j++) {
4             for (int k = j; k < n; k++) { k++
5                 System.out.println(i * j + k);
6             }
7         }
8     }
```

Design an algorithm for $\Theta(n^3)$.
RT(n^3) \propto $n^3 \cdot \log n$

Java docs

1. Read Java docs.

 @param param
 @return return
 @throws throws

 String m(=...){
 - - -
 }

3

 @precondition
 → assume

Unit Testing

1. Read unit tests.

2. No need to write
 Subject to test

3. Given problem
 come up with test cases.

Option 1:

```

    /**
     * @return ...
     * @param x ...
     * @param y ...
     * @throws X
     */
    double divide( double x, double y) throws X
    {
        if( y == 0) { throw new X(...); }

        return x / y;
    }

```

when user
tried to
call you!
about illegal value
passed by caller.

Option 2:

```

private double divide( double x, double y)
{
    return x / y;
}

```

assume:
 $y \neq 0$.

class MyClass {

```

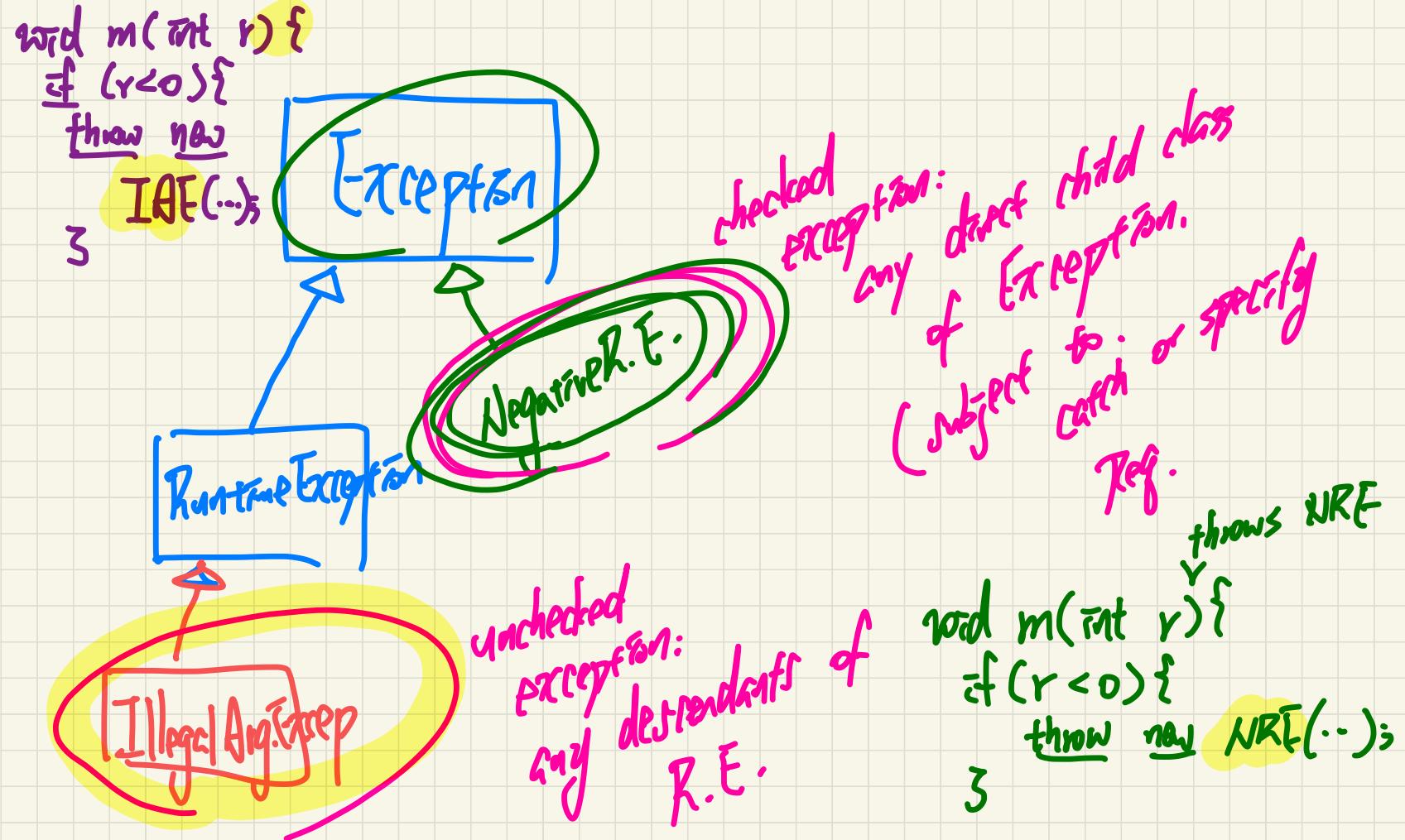
void m1() { divide(input, System.nanoTime()); }

private void divide( ... ) {
    ...
}

```

under what
circumstance
you call divide.

@precondition $y \neq 0$



Exercise

two counters

C1: min -1
max 3] init: 0

inc
C2: min 4
max 7] init: 4

dec

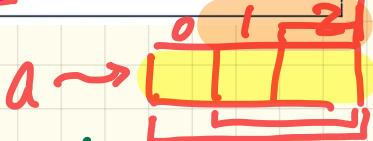
Correctness Proofs: Ideas

```
1 boolean allPositive(int[] a) { return allPosH(a, 0, a.length - 1);  
2 boolean allPosH(int[] a, int from, int to) {  
3     if (from > to) { return true; }  
4     else if (from == to) { return a[from] > 0; }  
5     else { return a[from] > 0 && allPosH(a, from + 1, to); } }
```

$\text{allPosH}(a, 0, 2)$
 $\hookrightarrow \text{allPosH}(a, 1, 2)$
 $\hookrightarrow \text{allPosH}(a, 2, 2)$
 $\hookrightarrow \text{allPosH}(a, 3, 2)$

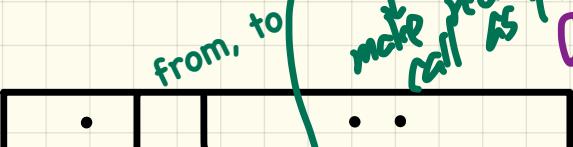
Base Case:

Empty Array



Base Case:

Array of Size 1

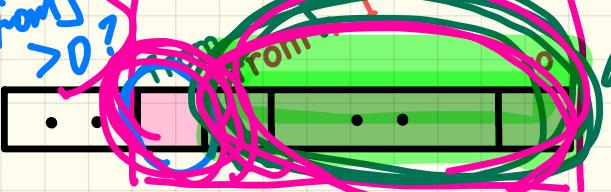


① Link to the Code
(loop #'s)

② Argue.

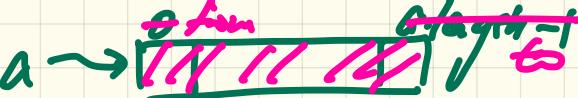
Recursive Case:

Array of size > 1



at [from+1]
at [from+2]
at [to]

Correctness Proofs



```
1 boolean allPositive int[] a { return allPosH (a, 0, a.length - 1);  
2 boolean allPosH int[] a, int from, int to) {  
3 if (from > to) { return true; }  
4 else if (from == to) { return a[from] > 0; }  
5 else { return a[from] > 0 && allPosH (a, from + 1, to); } }
```

I.H. true
if $a[from]$, $a[from+1], \dots, a[to]$ are all pos.

Via mathematical induction, prove that `allPosH` is correct:

Base Cases

- In an empty array, there is no non-positive number. \therefore result is **true**. [L3]
- In an array of size 1, the only one element determines the result. [L4]

Inductive Cases

- Inductive Hypothesis:** `allPosH(a, from + 1, to)` returns **true** if $a[from + 1], a[from + 2], \dots, a[to]$ are all positive; **false** otherwise.

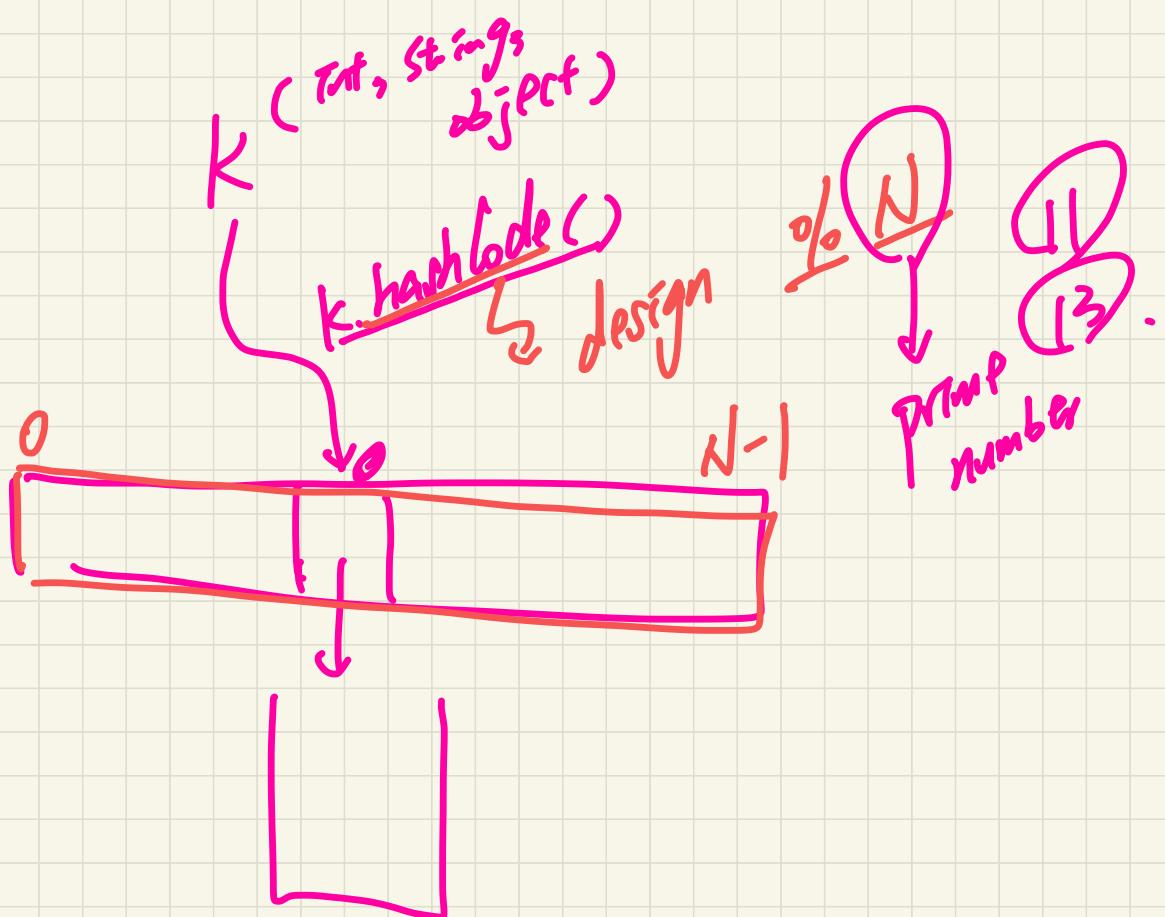
\Rightarrow `allPosH(a, from, to)` should return **true** if: 1) $a[from]$ is positive;
and 2) $a[from + 1], a[from + 2], \dots, a[to]$ are all positive.

- By **I.H.**, result is $a[from] > 0 \wedge \text{allPosH}(a, from + 1, to)$. [L5]

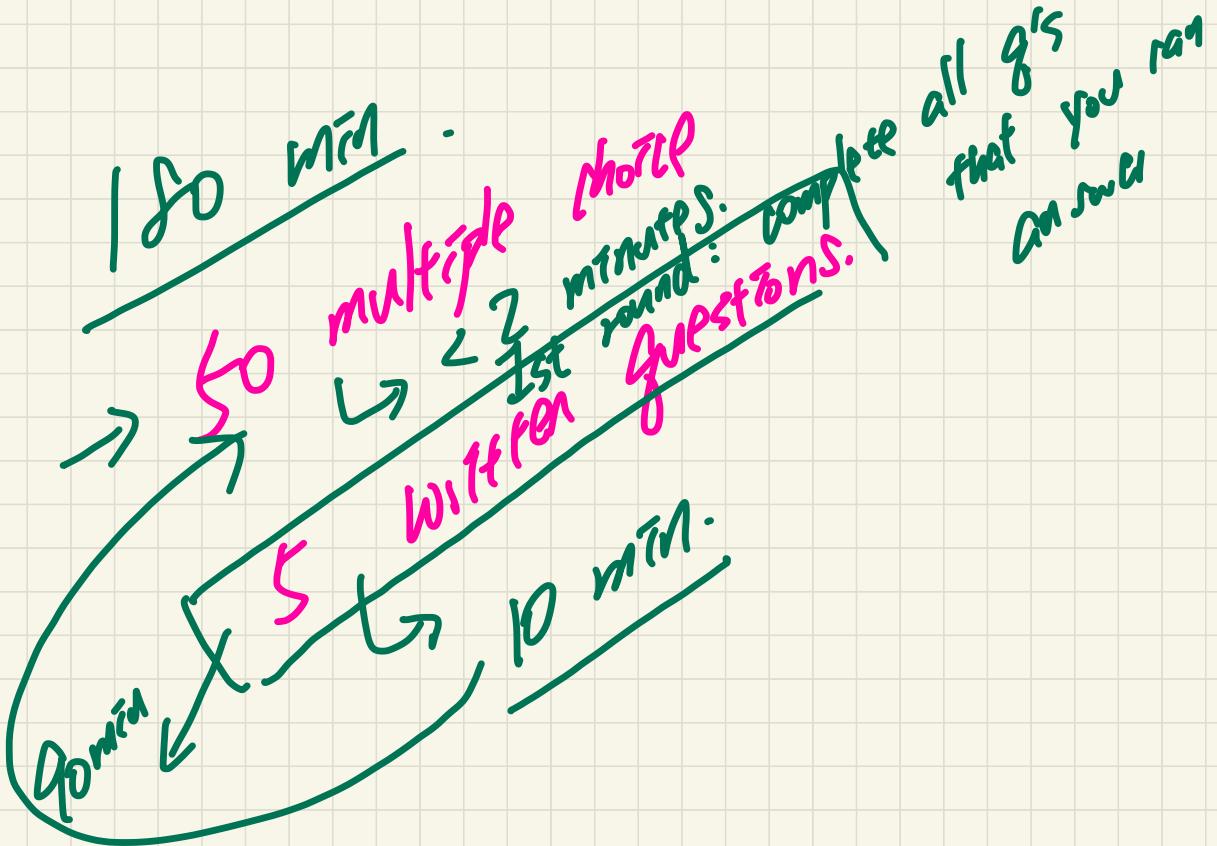
- `allPositive(a)` is correct by invoking `allPosH(a, 0, a.length - 1)`, examining the entire array. [L1]

base cases
empty case

1-element case



% \downarrow N
PRIME number
11
(13).

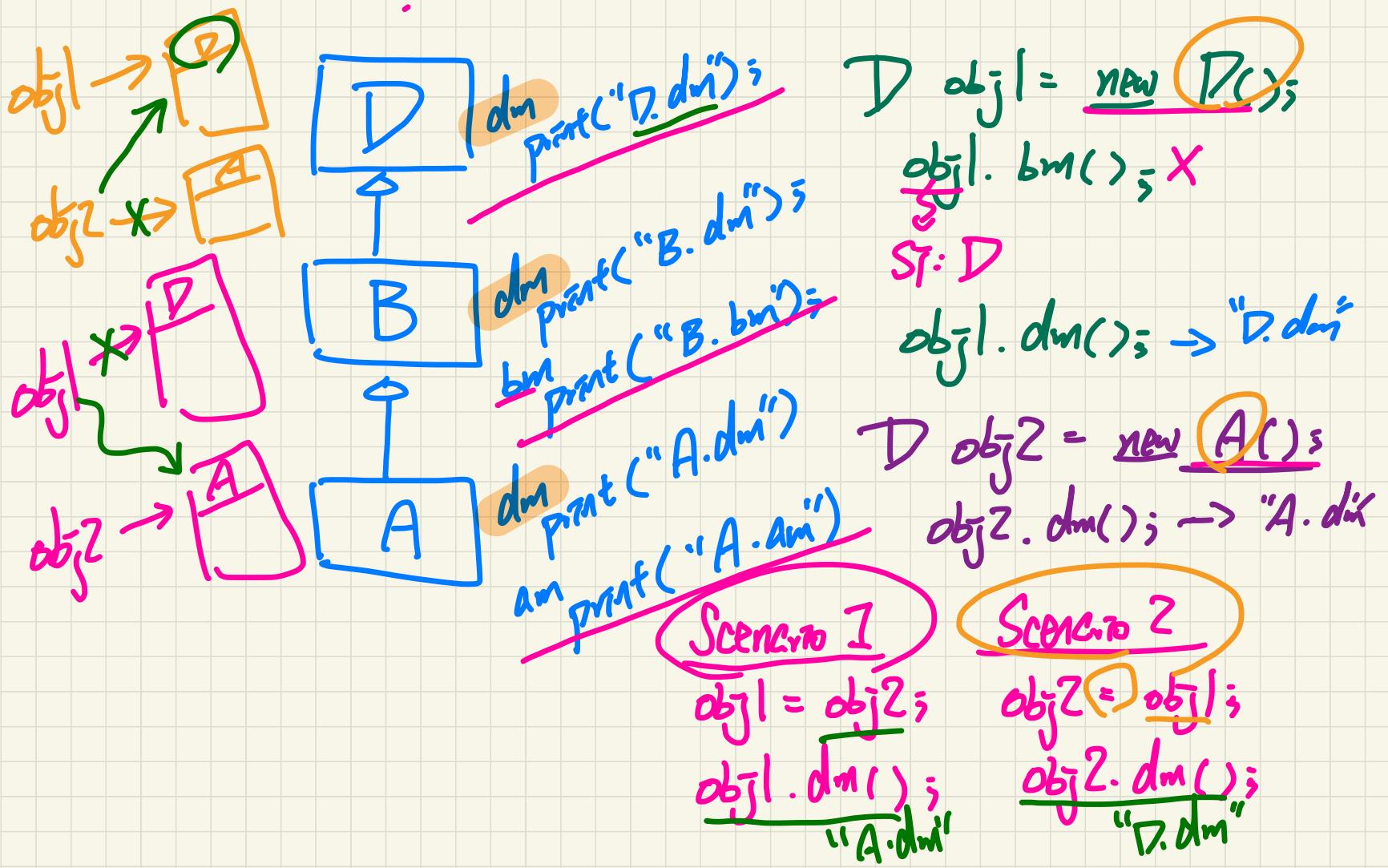


$$hc(k) = k \% 11$$

$$hc(k1) == hc(k2)$$

$$\underbrace{hc(23)}_1 \underset{==}{\circlearrowright} \underbrace{hc(34)}_1 \checkmark \quad \underline{\text{Collision}}$$

$$hc(23) == hc(\underbrace{23}_{23 \% 11}) \rightarrow \begin{array}{l} \text{jump right} \\ \text{for fsp} \\ \text{jump right.} \end{array}$$



```

class App {
    . . . main(~ ~) {
        C oc = new C();
        D obj1 = new A();
        oc.add(obj1, obj1);
        B obj2 = new A();
        oc.add(obj2, obj2);
        oc.add(obj2, obj2);
        b = obj2; ST: B
        (A) obj2; ST: B
        b = obj2; ST: B
        a = obj2; ST: A
    }
}

```

class C {
 [B] array;
 int noI; /* # of Items */
 void add(D d) {
 a[noI] = d; X
 ST: P
 b = obj2; ST: B
 void add(B b) {
 a[noI] = b; noI++;
 a[noI] = a; ST: A
 ST: B
 }
 }
}